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Robert E. Bainbridge MAI, SRA. MRICS
C-Store Valuations

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Toward a Better Understanding of Reilly's Law and the Trade Area of Convenience Stores

Robert E. Bainbridge MAI, SRA

This paper attempts to apply Reilly's Law to the trade area boundaries of convenience stores.

This paper is also an opportunity of exploration for this student to better understand Reilly's Law, its implementation, its limitations, and how it can be applied using criteria other than population.

Part 1 is an overview of Reilly's Law. Part 2 explores the original gravitation model and demonstrates through an example that different exponents in the denominator affect the conclusions of the model. Part 2 also explains the origin of squaring the distance in the denominator and shows that Reilly was incorrect to apply the Inverse Square Law from the original Newtonian Universal Law of Gravitation to the concept of retail trade.

Part 3 applies the concepts of the retail gravitation model to the convenience industry.

Part 1
Reilly's Retail Gravitation Model as Developed in the Current Literature

A convenience store, as used in this paper, is the business model developed in the 1990s of a retail enterprise that includes motor fuel sales, sales of high-turnover food and merchandise and a quick-serve restaurant offering or in-house prepared food program. This type of business is generally branded by national oil companies. Not included are the older service stations of the 1950s and 1960s, nor the grocery store retail channel.

The trade area of a convenience store remains an elusive concept, one more of generalization and rule of thumb, rather than a systematic or measurable analysis. Most owner-operators respond that the trade area of their stores is a two mile radius. For most of the last decade this has been the accepted and standardized view by this industry and it seems to have worked reasonably well for planning new store locations. However, during much of the latter part of the last decade more convenience stores were built than were demanded in the marketplace. The over-supplied market of today will see many stores close. Some industry analysts expect that six out of ten store will cease operations in the coming decade. With a problem of this magnitude, it is doubtful that the blame lies with any one cause. But, having said that, a better understanding of how trade areas are created and maintained in this industry could undoubtedly shed useful light on the relationship between supply and demand for convenience stores.

Reilly's Law[1] was developed in the 1930s and was a simple application of the Newtonian Universal Law of Gravitation to try to provide an explanation of how retail trade areas extended out from cities and towns. As with Newton's formulations for how the forces of gravity behaved, Reilly believed that larger cities have larger trade areas in direct proportion to their population and that the location of the boundary of the trade area between two cities was also inversely related to the square of the distance that separated them. The mathematical formula developed by Reilly is shown below.

[1]Reilly, William J., *Methods for the Study of Retail Relationships*, University of Texas Bulletin 2944 (Austin: University of Texas, 1929; reprinted 1959).

Formula 1
Reilly's Law of Retail Gravitation

$$D_1^2/D_2^2 = P_A/P_B$$

Where P_A and P_B are the respective populations of two cities, *A* and *B*, and D_A and D_B are their respective distances from the boundary.

The left side of the equation simply tells us where the boundary lies.

The right side of the equation is the factor or set of factors that determine the boundary. In Reilly's model, the population of towns has been commonly used on the right side of the equation. The common belief was that empirically at least, in Reilly's day, larger towns had more retail choices for consumers. For example, a rural family living midway between a small town and a large city in purchasing goods or services available in both towns, say bread and milk, would have no particular reason to prefer one town over the other and shopping trips devoted only to convenience items would tend to be equally divided between the two locations. If the trip included going to a movie and buying a suit of clothes, the preference would be for the larger town, because the larger town might have more than one movie theater and more clothing stores that the smaller town. Shopping trips of this sort will be directed predominately toward the larger town. Finally, for very specialized items, such as parts for a refrigerator or binoculars, that could not be purchased in the smaller town all shopping trips of this kind would be directed to the larger town.

Because shopping trips incur costs of time and money for the shopper, shopping trips tend to be consolidated into multi-purpose trips whenever possible. For this reason, the majority of shopping trips for a family located at the half-way point will be in the direction of the larger town because of the greater range of retail choices and activities.

According to Edgar M. Hoover, in *An Introduction to Regional Economics*[2],Reilly's Law

> *"...works reasonably well when tested against actual situations (which might be expected since it was derived empirically rather than theoretically), and has proved more durable than other "laws"."*

Reilly, a marketing specialist, called this boundary between two cites the "Break Point", designated as "BP". It was on this Break Point that trade was equally distributed between two cities. This formula implied that if two cities were of equal size, the Break Point was a straight line midway between them. If City A was twice as large as City B, then the trade area boundary between them, the Break Point, is 1.4 times as far from A and from B.

The formula for Reilly's "Break-Point" is a re-statement of Formula 1::

Formula 2
"Break Point" Formula

$$\text{BP} = \frac{\text{distance between city a and b}}{1 + \sqrt{\dfrac{\text{pop. b}}{\text{pop. a}}}}$$

The assumption that population size is the proxy for retail infrastructure may no longer be valid today. With the greater transportation efficiencies provided by the interstate freeway system, retail activity has moved away from small towns to more central locations, just as Christaller's[3] ideas about central place theory would suggest in a more mobile region where barriers to movement are now fewer than they were in the past.

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[2]Hoover, Edgar M., *An Introduction to Regional Economics*, Second Edition. Alfred A. Knopf, New York. 1975. Pp 126.

[3]Walter Christaller, in *Central Places in Southern Germany*, used deductive reasoning to explain on an empirical basis the settlement patterns in Southern Germany. He developed the pioneering concept of central place theory in the 19th Century..

Lloyd and Dicken[4] state that three things happen to a region when transportation linkages improve:

> 1. The spatial pattern of production has been transformed from a dispersed to a concentrated pattern.
>
> 2. There has been an increased differentiation between locations on the basis of their intrinsic qualities.
>
> 3. The degree of geographic specialization and the spatial extent of production have both increased.

Today, some small towns essentially have no retail base. So today, population alone is not the proxy for retail activity the way it once was prior to the freeway system.

[4]Lloyd, Peter E. and Peter Dicken, *Location in Space: A Theoretical Approach to Economic Geography*, Second Edition, Harper and Row, New York, NY. 1977. Pp190.

Part 2
Understanding the Gravitation Model

Although I was exposed to the Gravitation Model years ago in an undergraduate regional economics course, I never fully understood why the model was constructed the way it was. Most text books give scant attention to why the formula is constructed in its present form. Part of the reason for writing this paper was so that I could take the time and justify the effort of learning more about this subject.

One of the biggest mysteries to me as a student of regional economics was why the denominator (distance) was squared. What function did squaring perform? What difference would it make if the squaring function were left off? In the original gravitation model, the distance between the population centers is squared, as in the formula below[5].

Formula 3
Original Gravity Model

$$\frac{Pop_1 \times Pop_2}{distance^2}$$

The original gravity model, just as the Newtonian equation, attempts to measure the interaction between two places. This interaction is a function of the size of their respective populations and the distance that separates them. A series of examples will illustrate the practical effects of the model. To keep the math simple, small magnitude numbers are used.

Assume two towns with respective populations of 50 and 500.

A) If located 30 miles apart:

$$= \frac{50 \times 500}{30^2}$$

[5] The original gravity model was modified by Reilly into his "Break Point" formula shown as Formula 2.

$$= \frac{25{,}000}{900}$$

$$= 28$$

B) If located 15 miles apart:

$$= \frac{50 \times 500}{15^2}$$

$$= \frac{25{,}000}{225}$$

$$= 111$$

In other words, if the only change is that the distance is cut in half, then the effect is increased by a factor of nearly four.

C) If located 60 miles apart:

$$= \frac{50 \times 500}{60^2}$$

$$= \frac{25{,}000}{3{,}600}$$

$$= 7$$

In other words, if the distance is doubled, the attraction between the two towns decreases by a factor of, not two, but four.

D) If located 7.5 miles apart:

$$= \frac{50 \times 500}{7.5^2}$$

$$= \frac{25{,}000}{56.25}$$

$$= 444$$

So, if the distance is decreased by one-fourth, the effect is increased by almost 16 times.

E) If located 120 miles apart:

$$= \frac{50 \times 500}{120^2}$$

$$= \frac{25{,}000}{14{,}400}$$

$$= 1.74$$

So, if the distance is multiplied by four (4), the effect is reduced by a factor of slightly more than 16.

To summarize:

Distance	Nominal Effect	Effect on Relative Attraction
30 miles	Base	N/A
7.5 miles	Divided by 4	+16X
15 miles	Half	+4X
60 miles	Doubled	-4X
120 miles	Quadrupled	-16X

If the denominator in the gravity model is not squared, the calculations are as follows:

A) If located 30 miles apart:

$$= \frac{50 \times 500}{30}$$

$$= \frac{25{,}000}{30}$$

$$= 833$$

B) If located 15 miles apart:

$$= \frac{50 \times 500}{15}$$

$$= \frac{25{,}000}{15}$$

$$= 1{,}667$$

In other words, if the only change is that the distance is cut in half, then the effect is increased by a factor of two (2).

C) If located 60 miles apart:

$$= \frac{50 \times 500}{60}$$

$$= \frac{25{,}000}{60}$$

= 416

In other words, if the distance is doubled, the attraction between the two towns decreases by a factor of two (2).

D) If located 7.5 miles apart:

$$= \frac{50 \text{ x } 500}{7.5}$$

$$= \frac{25{,}000}{7.5}$$

= 3,333

If the distance is reduced by a factor of 16, the relationship increases by a factor of 4.

E) If located 120 miles apart:

$$= \frac{50 \text{ x } 500}{120}$$

$$= \frac{25{,}000}{120}$$

= 208

If the distance is increased by a factor of four (4), the relationship drops by a factor of four (4)

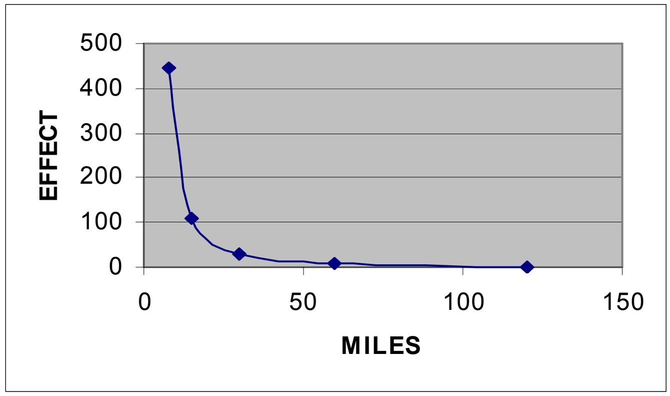
To summarize:

Distance	Nominal Effect	Effect on Relative Attraction
30 miles	Base	N/A
7.5 miles	Divided by 4	+4X
15 miles	Half	+2X
60 miles	Doubled	-2X
120 miles	Quadrupled	-4X

Assessing the Results

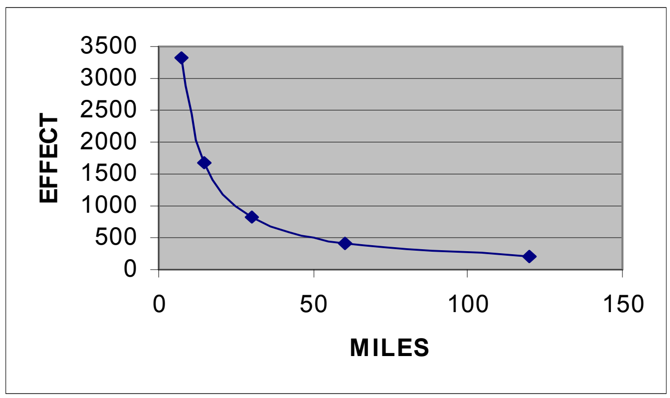
My initial impression was that squaring the denominator would have produced a curvilinear, or quadratic function. But, the shape of the function did nor change. Both the squared denominator and "un-squared" denominator produced curvilinear functions.

Figure 1 Effects of Squaring the Denominator



Squaring the denominator simply increases the magnitude change, as is seen in the y-scaling of the two graphs at right.

Figure 2 Effects of Not Squaring the Denominator



With this curvilinear pattern, the implication is that at the position of "zero" miles the effect would be infinitely high never crossing over the Y-axis. At an infinitely large distance the effect would be very small, but never "zero".

The practical purpose of the gravity model is to compare the relative interaction among pairs, or groups of cities. For example, in the area that I live in, does the larger city of Boise, Idaho have more of an effect on retail sales in Payette than the closer city of Ontario, Oregon? Payette, the town where I live, has a population 7,500. Boise is the largest city in the region. Boise has a population of 150,000 and is 60 miles from Payette. Ontario is a closer city of 12,000 and is located 6 miles from Payette. Do more Payette residents travel to Boise or to Ontario for shopping? The gravity model calculations are shown below.

Under the Squared Denominator Format

A) Retail Interaction with Payette and Boise

$$\frac{7{,}500 \times 150{,}000}{60^2} = \frac{1{,}125{,}000{,}000}{3{,}600} = 312{,}500$$

B) Retail Interaction with Payette and Ontario

$$\frac{7{,}500 \times 12{,}000}{6^2} = \frac{90{,}000{,}000}{36} = 2{,}500{,}000$$

Under the squared denominator format, Ontario has a much greater interaction with Payette than Boise. This implies that more of Payette's retail dollars are spent in Ontario than in Boise.

Under the Not Squared Denominator Format

A) Retail Interaction with Payette and Boise

$$\frac{7{,}500 \times 150{,}000}{60} = \frac{1{,}125{,}000{,}000}{60} = 18{,}750{,}000$$

B) Retail Interaction with Payette and Ontario

$$\frac{7{,}500 \times 12{,}000}{6} = \frac{90{,}000{,}000}{6} = 15{,}000{,}000$$

Here, when the denominator is not squared the results are actually different. Boise is calculated to have a greater interaction with Payette than Ontario. This implies that more of Payette's retail dollars are spent in Boise than in Ontario.

So, the squaring of the denominator is not just a scaling factor or insignificant part of the model's equation. The fact that the denominator is squared, or not, can actually produce different conclusions as to which trade exerts more influence. This is a fact that was not found or discussed in any of the literature I reviewed, which included the standard university text books on regional economic analysis.

Lloyd and Dicken[6] state on page 57 that the exponent of the denominator[7]

> *"...measures the frictional effect of distance."*

They further explain that the higher the value of the exponent (*b*), the greater the friction and therefore the more rapidly interaction falls off with distance. They state that John Q. Stewart argued that this exponent must be either one (1) or two (2), and to use any other number was *"...out of step with the model's physical derivation."* The impression left upon the student is that squaring the distance is relatively unimportant, at least any difference is not developed or expanded upon.

What Lloyd and Dicken fail to explain to the reader is that the selection of this exponent, whether it be one (1), two (2) or three (3), can produce differing conclusions.

It is intuitive that the greater the measured friction of distance (the exponent), the less interaction there is between the cities. However, to me at least, it is not intuitive that the calculated result could be altered so much as to reach a different conclusion. This tells me that the selection of the exponent will be critical to the conclusion. Even in light of Stewart's comments, the exponent could not only be one (1) or two (2), but any fraction in between, such as 1.25. 1.5, or 1.75. Does it make a difference? Yes, and much more so than has been implied in the literature.

How does the analyst know whether the exponent is one (1), two (2) or some number in between? Since Reilly's model was derived empirically, there is no way to tell.

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[6]Ibid.

[7]So far in our discussion this has been "2". Lloyd and Dicken call this *b*.

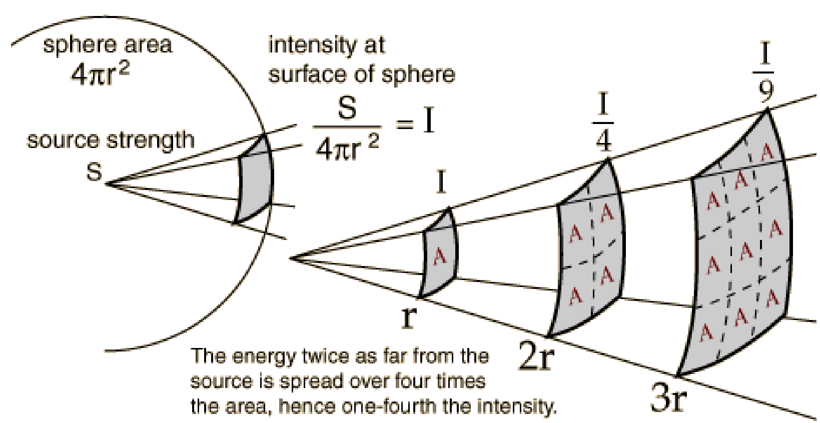
Influence of the Inverse Square Law on Reilly

Most models assume the exponent is two (2) with very little explanation as to why that number is being used, other than "that's the way it has always been". In historical reality, the exponent has commonly been two (2) because that is what Newton used in 1687. It is apparent that the literal gravitational attraction between objects in space might be more or less than the retail attraction between cities given that some cities are connected by interstate freeways and other infrastructure and some are not, or at least not to the same degree.

The origins of squaring the denominator lies with Keppler's Third Law and Newton's Universal Law of Gravitation[8]. The reason for squaring the denominator in Newton's original application is strictly geometric in its origin. Any point source which spreads its influence in all directions without limit to range will obey the Inverse Square Law. This was a concept put forth by Edmund Halley at Oxford and proofed and published by Sir Isaac Newton in the Universal Law of Gravitation. In 1931, William J. Reilly adapted Newton's original formula to explain the relationship between retail trade areas.

The geometric concept of the Inverse Square Law is shown in the diagram below.

Figure 3 Visual Depiction of the Inverse Square Law



Moving away from the point source causes the influence to decline by an amount equal to the square of the distance. The squared relationship is due to the original assumption that the point source spreads out in all directions and therefore the influence is defined by the area of a sphere.

The Inverse Square Law is correct for all influences that originate from a point source and spread out in all directions, such as planetary gravity, light, radiant energy, and electricity.

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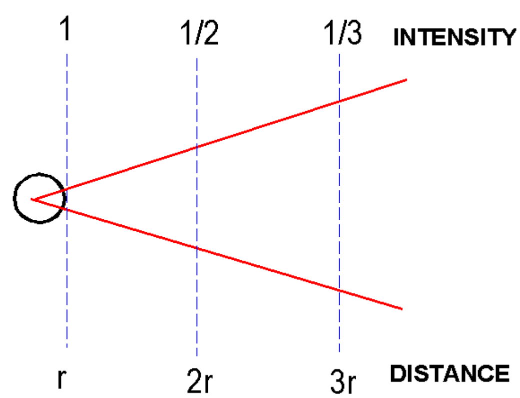
[8]Newton, Sir Isaac, *Philosophiæ Naturalis Principia Mathematica.* 1687.

However, Reilly was wrong to apply the Inverse Square Law to retail trade concepts. Reilly's application of the Inverse Square Law to retail trade inherently assumes that the influence of the trade area is spreading out in all directions in a three-dimensional environment, as shown in Figure 3. For all practical purposes, retail trade takes place in a two-dimensional environment. Width and length play a role on the surface of a plane. But, height and depth have no practical significance in our retail world. The influence of any trade area is not felt above the surface of the earth in the sky, nor beneath the surface of the earth underground. Therefore, the assumption about the distance decay of influence should more correctly be related to the area of a circle, rather than the area of a sphere.

As the diagram below shows, this changes the gravitation model formula from one of squaring the denominator to leaving the denominator "un-squared". The reason is simply one of geometry, should we calculate a sphere of influence, or a circle of influence?

Figure 4 Influence of a Circle



The answer is a circle.

We have already seen that squaring the denominator or not squaring the denominator does indeed change the conclusion of the model. In our example of the influence of Boise or Ontario on retail trade in Payette, squaring the denominator indicated that Ontario had more influence on Payette's retail trade. However, when the denominator was not squared Boise had the greater influence.

My conclusion in this paper is that Reilly's Law, as it has been historical developed and applied, under-estimates the influence of a trade area, especially large, distant trade areas, because the denominator should not be squared. In other words, the influence of a trade area extends for a longer distance and is more intense at any given interval than what Reilly's Law has previously implied.

Formula 3
Original Gravity Model

$$\frac{Pop_1 \times Pop_2}{distance^2}$$

The denominator in the original gravity model, as shown at right and published in most text books on the subject, should not be squared.

Part 3
Understanding the Micro Trade Area

However, Reilly had a good idea that proved to be remarkabley accurate in his day for explaining consumer buying choices and trade area boundaries. But, when population no longer can be a proxy for retail sales activity, other more practical measurements, such as retail space square footage or number of retail establishments may be selected for the right side of the equation.

Reilly's model need not apply only to general retail activity. His ideas can be adapted to analyze specific types of retail trade, such as grocery stores, motels, or retail fuel properties.

Formula 1
Reilly's Law of Retail Gravitation

$$D_1^2/D_2^2 = P_A/P_B$$

Also, the right side of Reilly's original equation is not constrained to physical attributes, the way Reilly conceptualized his model. Nothing prevents the analyst from choosing economic characteristics for the right side of the equation. Indeed, today when competition within a retail channel is so intense and one competitor appears pretty much the same as another, something other than physical characteristics should be used. In other words, if the competing retail establishments with a specific channel are all about the same size, the same color and the same age, then no reason exists to believe the consumer is making the shopping decision based upon physical attributes. The choice is more likely related to the purchase price of the goods or services, when all other factors are indistinguishable. Indeed, the single largest visual element on most convenience store advertizing signs is the retail price of gasoline. Today, store operators must intuitively expect that price is one of the most important criteria in attracting customers.

So, why not use retail price for the right side of the equation?

In the context of applying retail price into the right side of Reilly's equation, we can illustrate the concept using the example of a WalMart store that has entered the trade area of a convenience store. Typically, WalMart retails motor fuel at a price far below the street price of most convenience store operators. This retail price differential is so great that WalMart's competitors have cried foul and have petitioned the government to enact below cost selling legislation. So far, about half the states have enacted below cost selling legislation and have made it illegal for big box mass merchandisers to sell gasoline below cost.

How do retail price differentials affect the trade area of a convenience store? We can use Reilly's "Break-Point" formula, which he adapted from Newton's Universal Law of Gravitation, to answer the question. The classic "Break-Point" formula is shown at right.

Figure 5 Original "Break-Point" Formula

$$BP = \frac{\text{distance between city a and b}}{1+\sqrt{\dfrac{\text{pop. b}}{\text{pop. a}}}}$$

To modify this formula for retail price instead of "population", the formula below will be used. Assume that WalMart's posted retail price for regular, unleaded gasoline is $1.50 per gallon. The nearest competitive gasoline retailer is a convenience store located one-half mile away and their posted retail price is $1.65. Holding all other factors constant, if both retailers posted the same price, then the trade area boundary would be exactly half way between the two locations, or at the one-quarter mile position. Using Reilly's "Break-Point" formula in Figure 5, as would be expected, the price differential causes the "Break-Point" of the trade area to move farther away from the lower-priced WalMart location. In other words, WalMart's trade area expanded because they are selling gasoline at a lower price.

1) Using Reilly's Original "Break-Point" Formula

$$\mathbf{BP} = \frac{.5 \text{ miles}}{1 + \sqrt{(1.50/1.65)}}$$

$$= \frac{.5 \text{ miles}}{1 + \sqrt{0.9090}}$$

$$= \frac{.5 \text{ miles}}{1 + .9534}$$

$$= 0.2559 \text{ miles}$$

The differential in retail prices expands WalMart's trade area by 2.4% (.0059/0.25) and reduces the competitor's trade area by the same amount using the squared denominator derived from the Inverse Square Law, as developed by Reilly in his model.

The more correct procedure is to not square the denominator. The new calculations as they apply in the "Break-Point" formula are shown below.

2) Using the "Un-Squared" Adaptation of Reilly's "Break-Point" Formula

$$BP = \frac{.5 \text{ miles}}{1 + (1.50/1.65)}$$

$$= \frac{.5 \text{ miles}}{1 + 0.9090}$$

$$= 0.2619 \text{ miles}$$

The differential in retail prices expands WalMart's trade area break point with this competitor by 4.8% (.0119/0.25) using the "un-squared" denominator.

WalMart must decide if a 10% reduction ($1.50 per gallon v. $1.65 per gallon) in retail price is worth a 10% gain in market trade area with this competitor[9]. Under Reilly's original model the trade area gain would have appeared to have been only half this amount, or about 5.0%.

[9]To use only the gain in WalMart's trade area as measured by the "Break-Point" would preclude the fact that with WalMart's geographic gain, someone else lost. In other words, the competitor's trade area boundary receded at the same time by the same amount. So, the cumulative effect on the market is doubled. WalMart captured 10% more of the total trade area.

Formula 3
Original Gravity Model

$$\frac{Pop_1 \times Pop_2}{distance^2}$$

Going back to the original gravity model, we can check the total influence on the trade area resulting from the retail price differential.

The original formula appears at right. Substituting the retail prices for gasoline and the distance used in the previous example results in the following calculations for total influence in the trade area:

1) Reilly's Original Gravitation Model Using the Squared Denominator

$$\textbf{Influence} = \frac{1.50 \times 1.65}{\sqrt{.5}}$$

$$= \frac{2.475}{.707}$$

$$= 3.5$$

The magnitude of influence in the trade area is calculated here at 3.5.

Now, we will perform the same calculations without squaring the denominator.

2) Modified Gravitation Model
Not Squaring the Denominator

$$\textbf{Influence} = \frac{1.50 \times 1.65}{.5}$$

$$= \frac{2.475}{.5}$$

$$= 4.95$$

Without squaring the denominator, the influence of WalMart's reduced pricing in this market, where WalMart's retail gasoline price is $1.50 per gallon and the prevailing street price is $1.65 per gallon, is 41% higher (4.95/3.5) than the original gravitation model would have implied. That is a significant difference.

Conclusion

Squaring the denominator in Reilly's Law of Retail Gravitation arbitrarily underestimates the influence of the dominant trade area, both with respect to distance and intensity at any given interval.

More accurate results are obtained when the denominator is not squared because this approach most nearly reflects the geometry of the retail environment.

Are the differences in the two methodologies significant?

Yes, here we have shown that the estimated trade area of a large competitor is substantially greater than what Reilly's original retail gravity model would have suggested.

Toward a Better Understanding of Reilly's Law and the Trade Area of Convenience Stores

Robert E. Bainbridge MAI, SRA

151 S.W. 1st Street
Ontario, OR 97914
(541) 823-0029 VOICE
(541) 823-0079 FAX
bbmw@fmtc.com

PREPARED FOR:

Dr. Thomas Hamilton, Ph.D.
Assistant Professor of Real Estate
Department of Finance, College of Business
University of St. Thomas